# EE 508
# Lecture 27

Nonideal Effects in Switched Capacitor Circuits

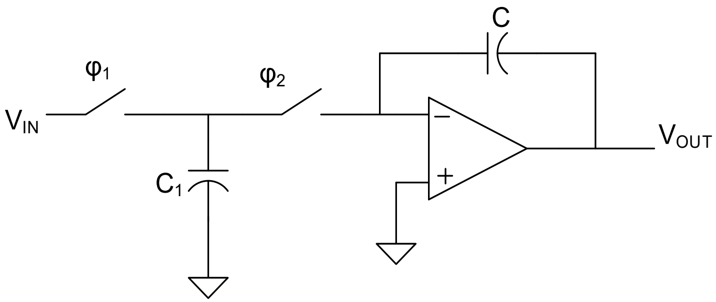
- Noise

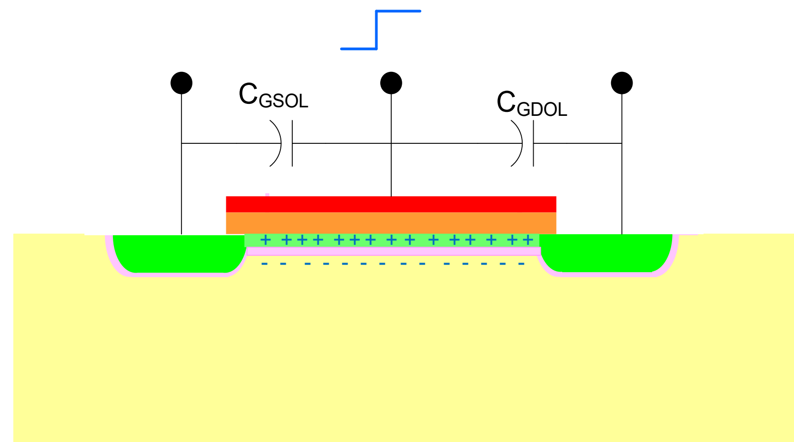
Switched-Resistor Filters

Other Integrators

Review from Last Lecture

# Charge Injection



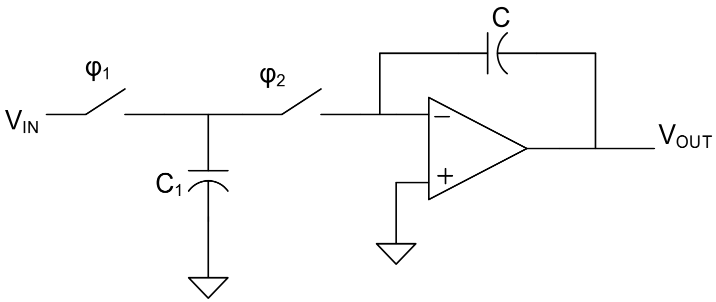



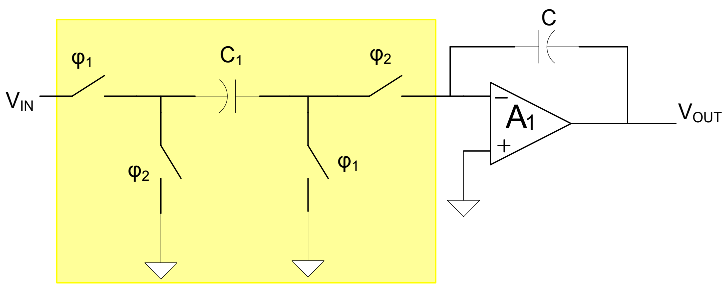

n-channel MOSFET

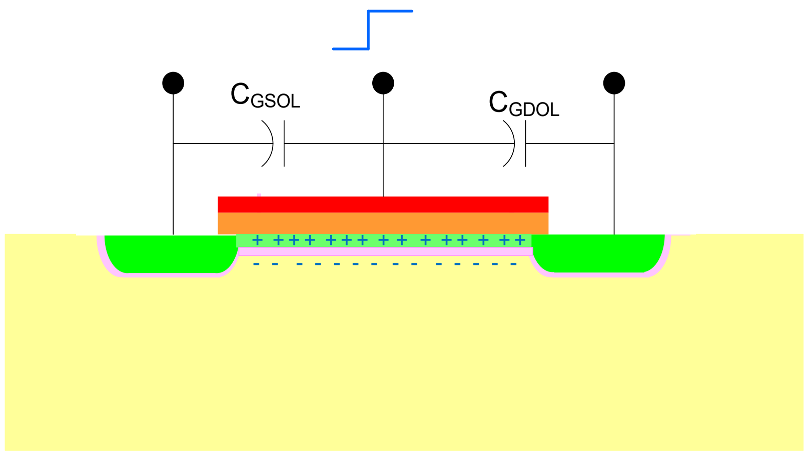
- If $\Phi_1$ opens slowly, channel charge will all exit through $V_{IN}$
- If $\Phi_1$ opens quickly, some charge will exit to left and some to right and split depends upon impedances seen to left and right
- Often not practical to open switches slowly
- Channel charge injection introduces errors in charge transfer and affects linearity

Review from Last Lecture

# Charge Injection



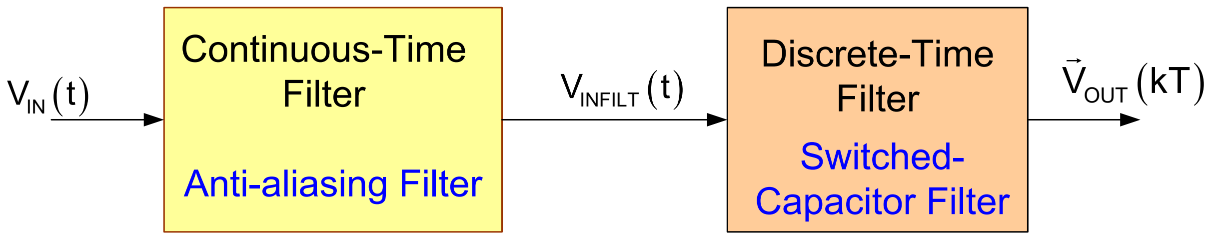



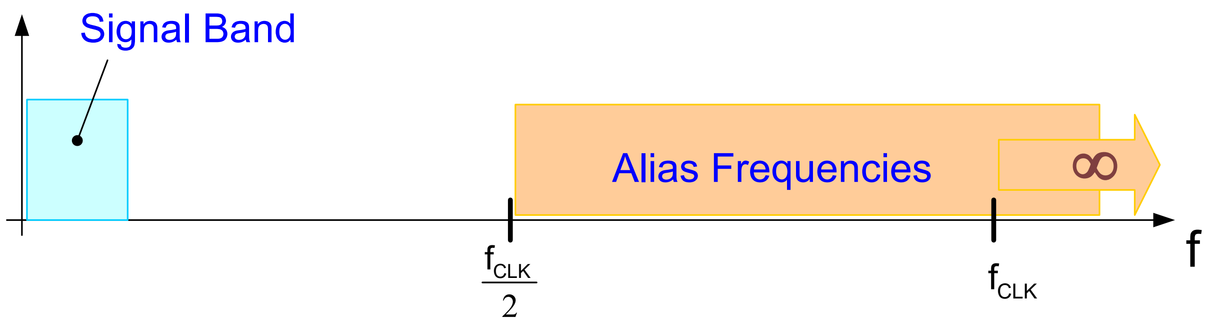




n-channel MOSFET

- Somewhat more complicated in multi-switch implementations
- There will naturally be a small amount of skew on clocks and this skew will affect charge injection
- Charge injection from some switches can be reduced or eliminated by using advanced clock (e.g. lower $\Phi_1$ switch opens before upper $\Phi_1$ switch)

# Anti-aliasing filter often required to limit frequency content at input to SC filters




Why not just make the clock frequency >> signal band edge ?

Recall in the continuous-time RC-SC counterparts

$$f_{POLES} \cong \frac{1}{RC} \cong f_{CLK} \, {C_1}/{C}$$

Since $f_{POLES}$ will be in the signal band (that is why we are building a filter) large $f_{CLK}$ will require large capacitor ratios if $f_{CLK} >> f_{POLES}$

- Large capacitor ratios not attractive on silicon (area and matching issues)
- High $f_{CLK}$ creates need for high GB in the op amps (area,power, and noise increase)

Often $f_{CLK}/f_{POLES}$ in the 10:1 range proves useful (20:1 to 5:1 typical)

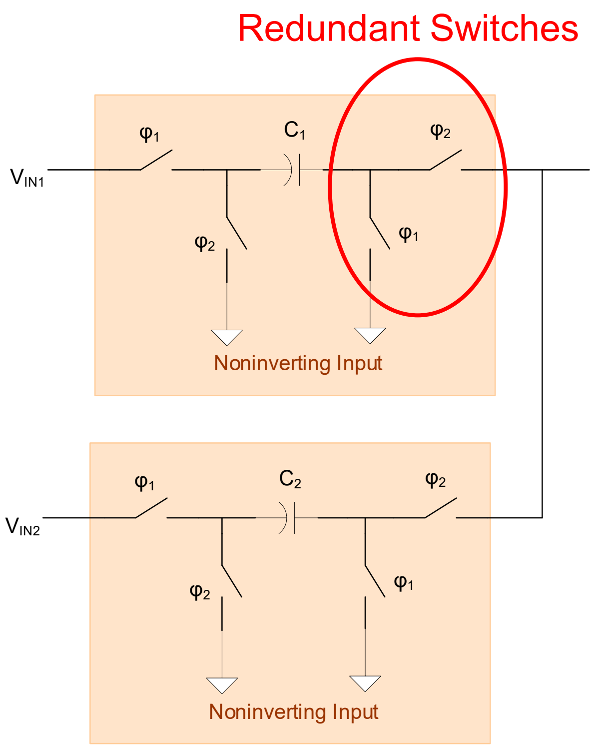
Review from Last Lecture

# Nonideal Effects in Switched Capacitor Circuits

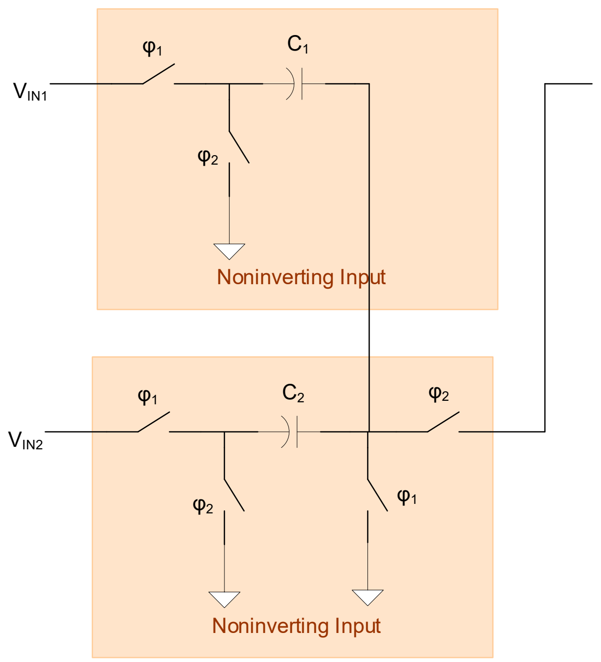
- Parasitic Capacitances
- Charge Injection
- Aliasing
- → Redundant Switch Removal
- Matching
- Noise

# Review from Last Lecture

# Elimination of Redundant Switches




Switched-Capacitor Input
with Redundant Switches




Switched-Capacitor Input with
Redundant Switches Removed

Although developed from the concept of SC-resistor equivalence, SC circuits often have no Resistor-Capacitor equivalents

Review from Last Lecture

# Nonideal Effects in Switched Capacitor Circuits

- Parasitic Capacitances
- Charge Injection
- Aliasing
- Redundant Switch Removal
- Matching
- Noise

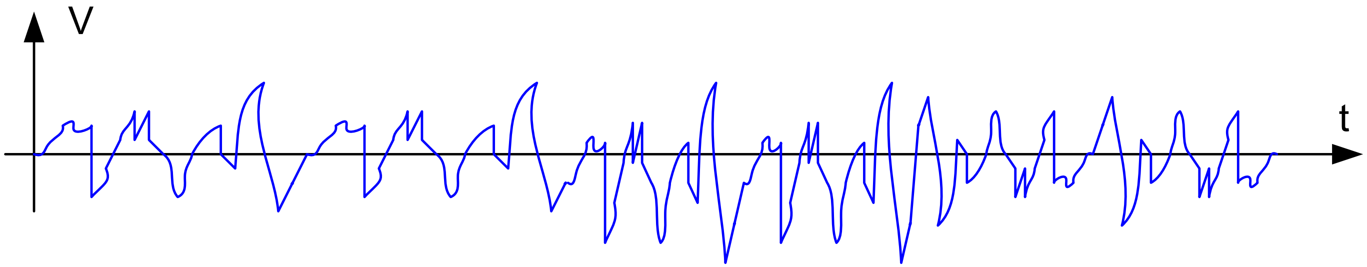
# Matching

- Matching is a statistical concept and directly relates to yield
- With good layout, matching to 0.01% or better can be achieved
- Common-centroid widely used to eliminate gradient effects
- Pelgrom parameter useful for analytically predicting yield with common-centroid layouts
- Area affects local variations
- Little in the literature or in PDKs to predict matching without gradient cancellation
- Must match all contacts and interconnects to get good matching
- Neighbor effects are important

# Nonideal Effects in Switched Capacitor Circuits

- Parasitic Capacitances
- Op Amp Affects
- Charge Injection
- Aliasing
- Redundant Switch Removal
- Matching
- Noise

# Noise in Continuous-time Linear Systems




Noise in continuous-time systems

$$v_{RMS} = \sqrt{\lim_{T_x \to \infty} \left( \frac{1}{T_X} \int_0^{T_x} v_n^2(t)dt \right)}$$

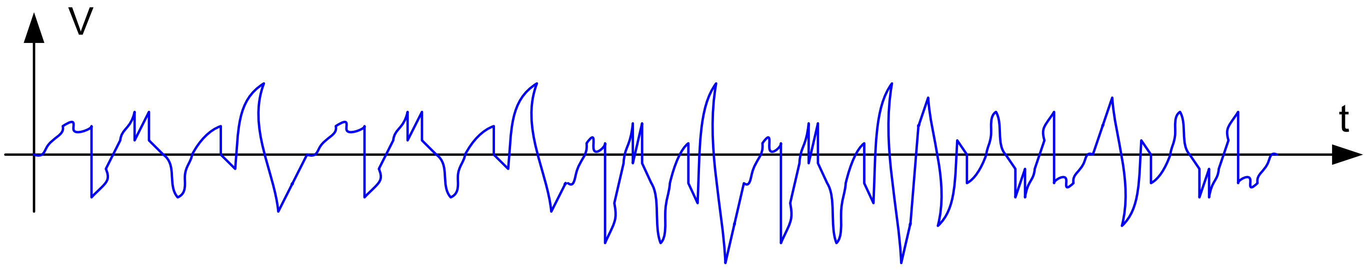
$\int_0^{T_x} v_n^2(t)dt$ is a random variable

At the design phase, we do not know what the time-domain noise characteristics will be

$$v_{RMS} = E\left( \sqrt{\lim_{T \to \infty} \left( \frac{1}{T} \int_0^{T} V^2(t)dt \right)} \right)$$

E denotes the expected value of the random variable

# Noise in Continuous-Time Linear Systems




Noise often characterized by the spectral density S

$$\mathcal{v}_{RMS} = \sqrt{\left( \int_0^{\infty} S(f)df \right)}$$

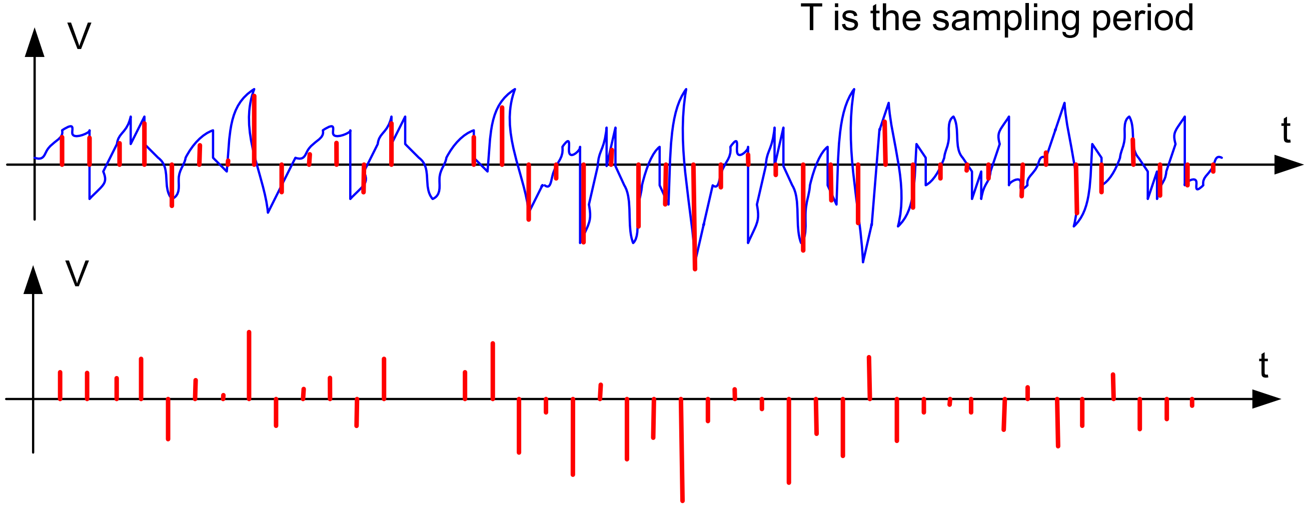
$$\int_{f=0}^{\infty} S(f)df = \lim_{f_x \to \infty} \int_{f=0}^{f_x} S(f)df$$

Thus

$$\sqrt{\lim_{T_x \to \infty} \left( \frac{1}{T_X} \int_0^{T_x} \mathcal{v}_n^2(t)dt \right)} = \sqrt{\left( \int_0^{\infty} S(f)df \right)} = \sqrt{\lim_{f_x \to \infty} \int_{f=0}^{f_x} S(f)df}$$

# Relation between continuous-time and discrete-time noise

Assume $v_n(t)$ is a continuous-time noise source
T is the sampling period




## Noise in discrete-time linear systems

$$\vec{V}(kT) = \langle v_n(mT) \rangle_{m=0}^{\infty}$$

$$\vec{V}_{RMS} = \sqrt{\lim_{M\to\infty}\left(\frac{1}{M}\sum_{k=0}^{M} v_n^2(kT)\right)}$$

$$\sigma\left(v_n(kT)\right) = v_{RMS}$$

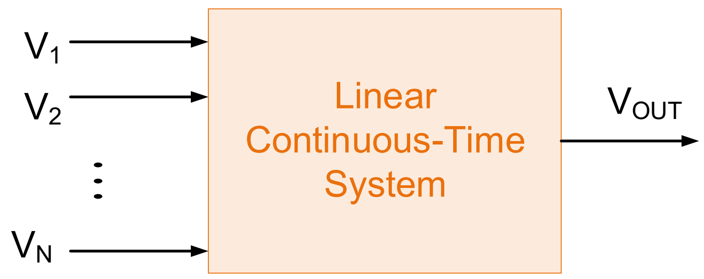
## Noise in continuous-time linear systems

$$v_{RMS} = E\left(\sqrt{\lim_{T\to\infty}\left(\frac{1}{T}\int_0^T V^2(t)dt\right)}\right)$$

Noise in discrete-time system is identical to that of the continuous-time system

$$\vec{V}_{RMS} = v_{RMS}$$

# Noise Summing in continuous-time Linear Systems



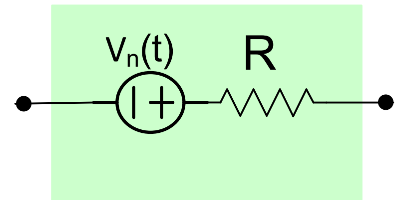

$$V_{OUT}(s) = \sum_{i=1}^{N} T_i(s) V_i(s)$$

$$S_{OUT} = \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2$$

$$\mathcal{V}_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \left|T_i(j\omega)\right|^2 df}$$

# Noise

- Capacitors do not contribute any noise
- Resistors contribute thermal noise
- Switches contribute thermal noise




Noise model of any resistor
$V_n(t)$ dependent on value of R

$$S_{Vn} = 4kTR$$

- Noise due to switches looks like “capacitive” noise

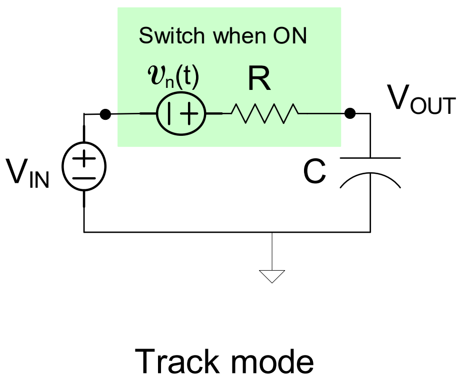
$$V_{RMS} = \sqrt{\frac{kT}{C}}$$

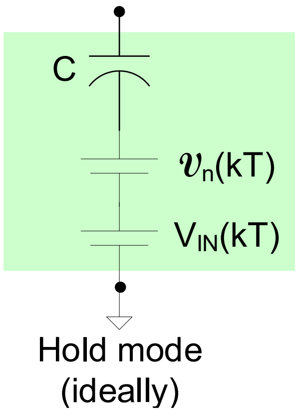
# Noise in switched capacitor circuits

- Noise due to switches looks like "capacitive" noise

$$V_{RMS} = \sqrt{\frac{kT}{C}}$$

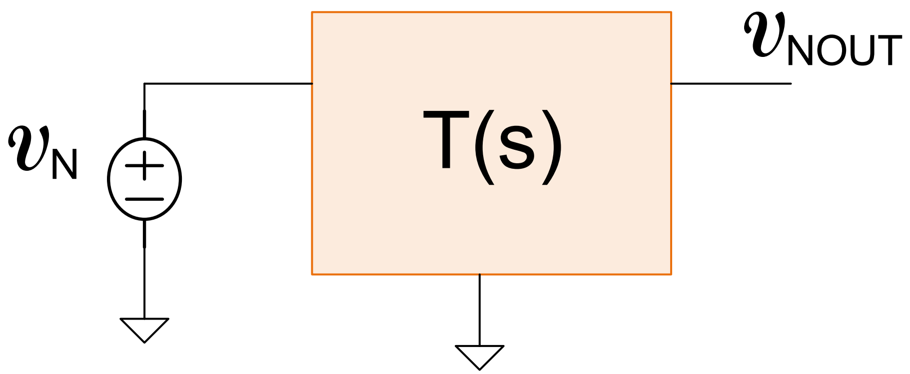
Consider a capacitor that is sampling an input signal $V_{IN}$




Neither the noise voltage nor the input voltage sampled onto C will be ideal if R is not near 0Ω

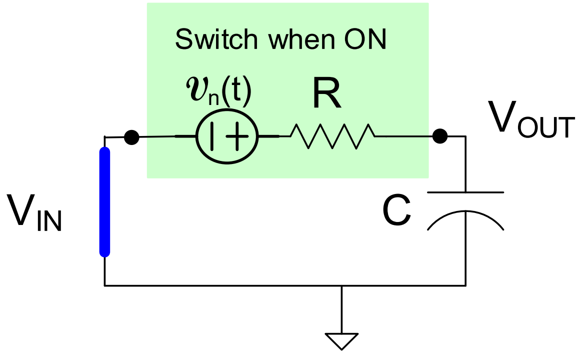
# Noise review




Component of spectral density at output due to any noise source with spectral density $S_X$ given by

$$S(\omega) = S_X \left| T(j\omega) \right|^2$$

# Noise during sampling phase



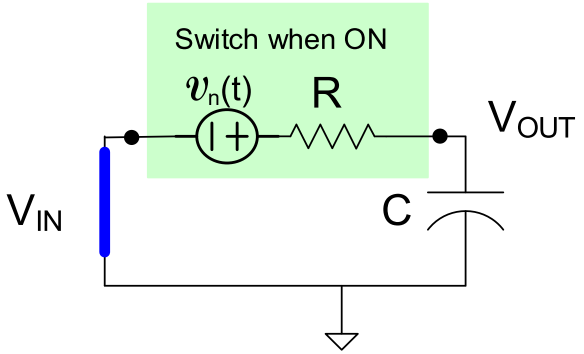

$$S(\omega) = S_X \left| T(j\omega) \right|^2$$

$$T(s) = \frac{1}{1 + RCs}$$

$$S_{Vn} = 4kTR$$

$$S_{VOUT} = 4kTR\left( \frac{1}{1+(RC\omega)^2} \right)$$

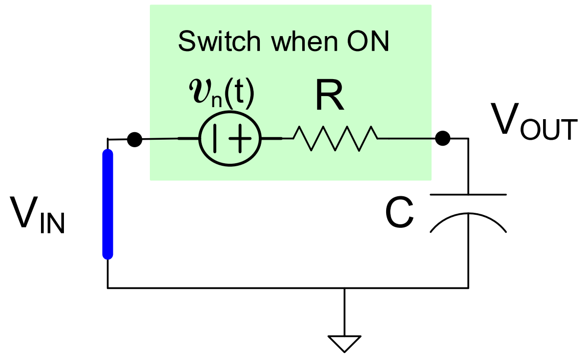
# Noise during sampling phase




$$S_{VOUT} = 4kTR\left(\frac{1}{1+(RC\omega)^2}\right)$$

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\,df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2R^2C^2}\,df}$$

# Noise during sampling phase

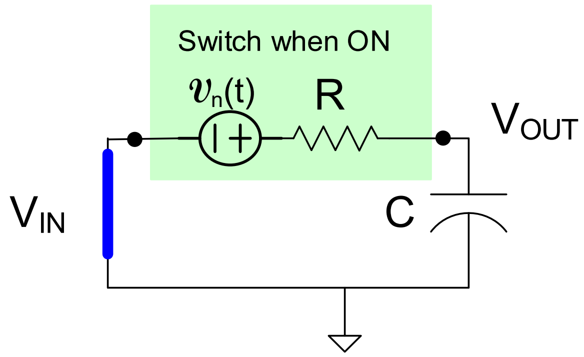



$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\,df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2R^2C^2}\,df}$$

It can be shown that this integral is independent of R and is given by

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\,df} = \sqrt{\frac{kT}{C}}$$

# Noise during sampling phase



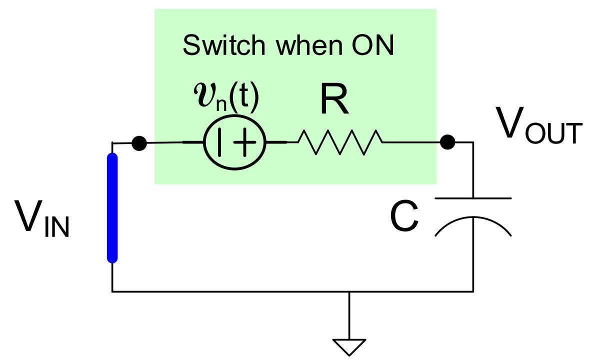

$$v_{n_{RMS}} = \sqrt{\frac{kT}{C}}$$

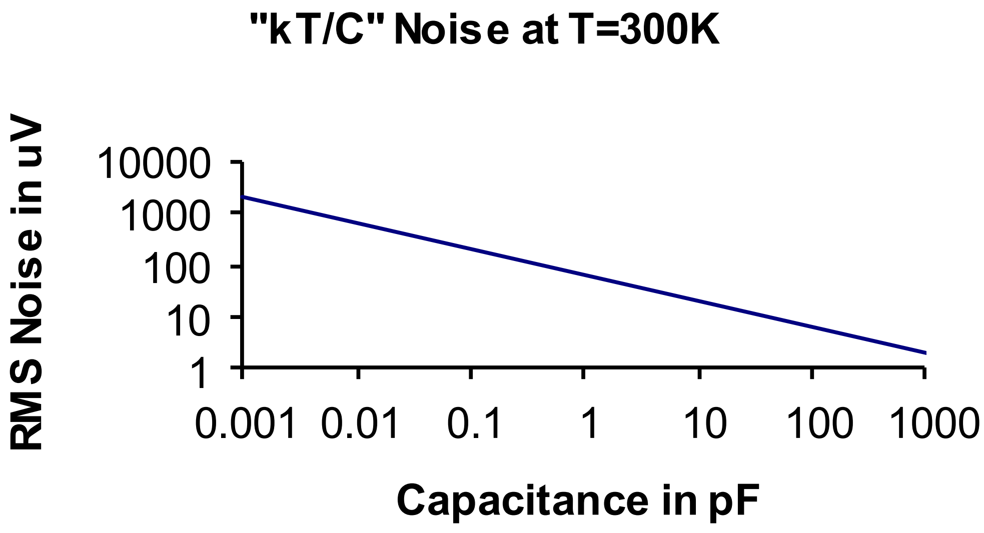
RMS noise voltage on C is independent of the state of the switch

So sampled RMS noise voltage should be same as instantaneous RMS voltage

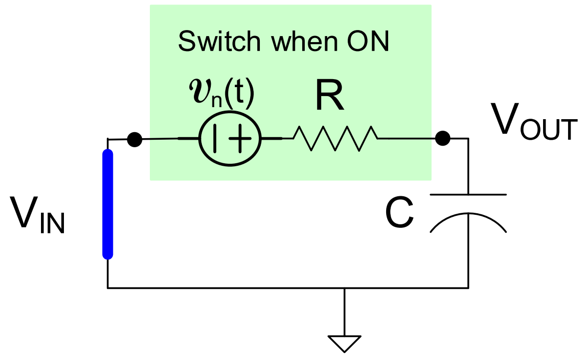
Highly temperature dependent

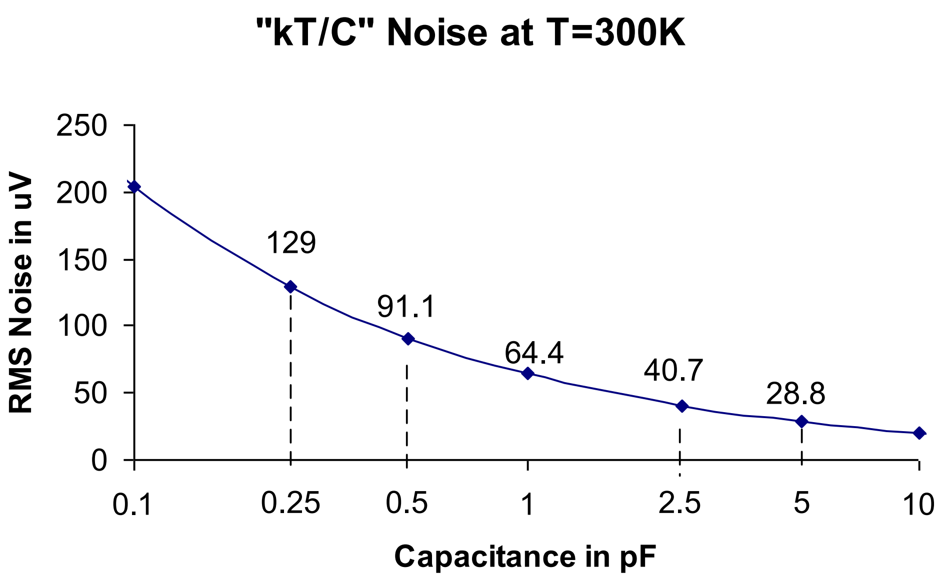
# Noise during sampling phase



# Noise during sampling phase



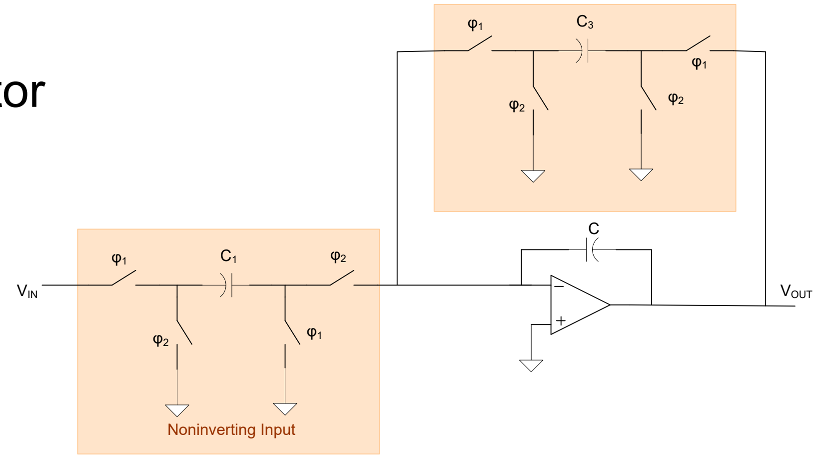
# Noise

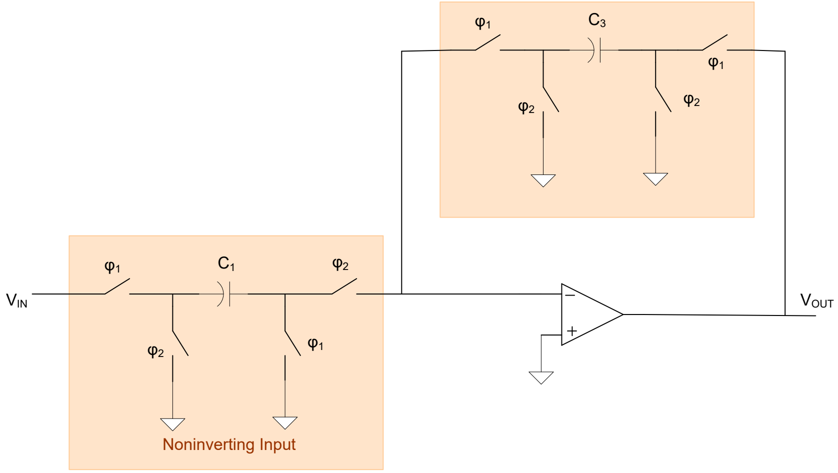
- Capacitors do not have any noise source
- Switches contribute thermal noise
- Noise due to switches looks like "capacitive" noise $V_{RMS} = \sqrt{\frac{kT}{C}}$

Be careful with calculating noise in SC circuits !

# Switched Capacitor Amplifiers

Elimination of the Integration Capacitor

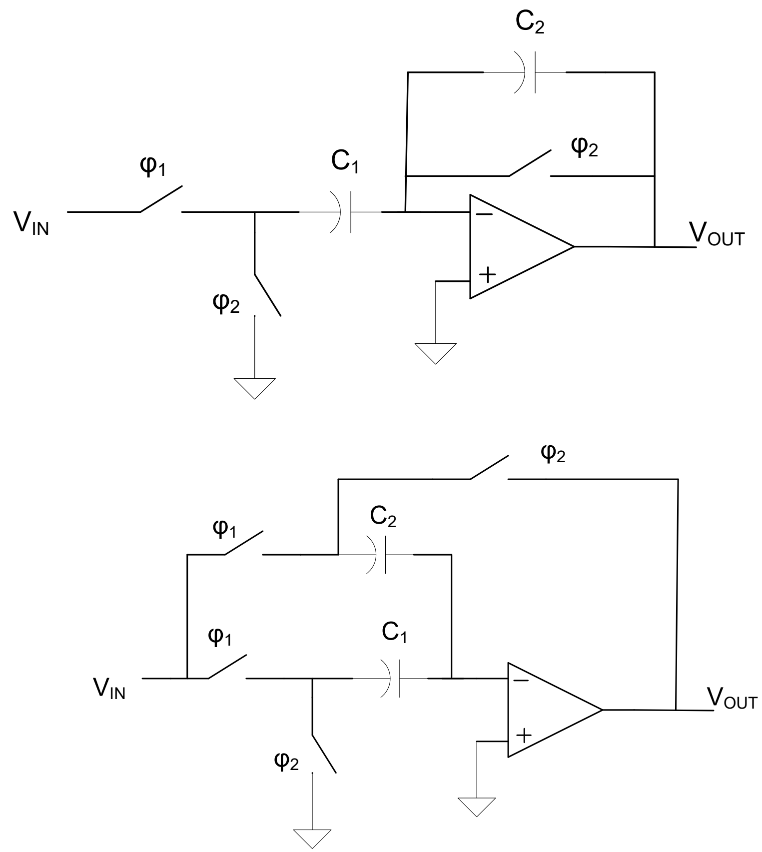



What happens if the integration capacitor is eliminated?

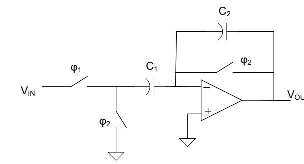
- Serves as a SC amplifier with gain of $A_V=C_1/C_2$
- SC amplifiers and SC summing amplifiers are widely used in filter and non-filter applications

# Switched Capacitor Amplifiers

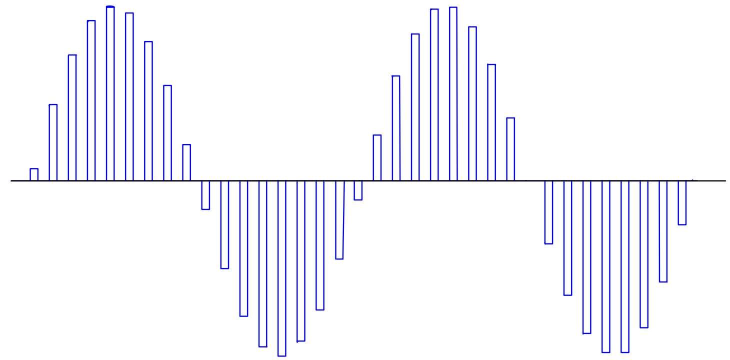


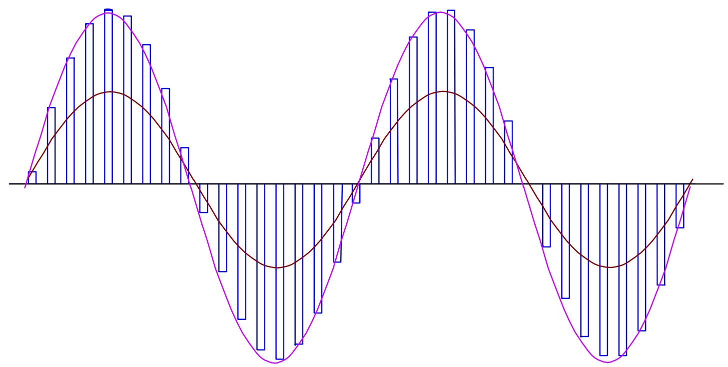

- Summing, Differencing, Inverting, and Noninverting SC Amplifiers Widely Used
- Significant reduction in switches from what we started with by eliminating C in SC integrator
- Must be stray insensitive in most applications
- Outputs valid only during one phase

# Switched Capacitor Amplifiers

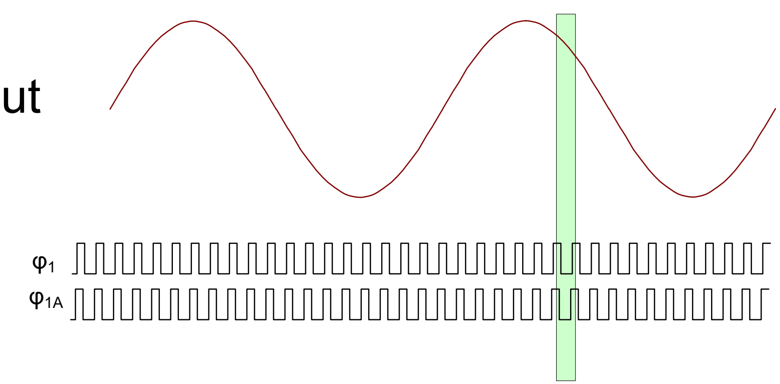



Output





Input



# Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

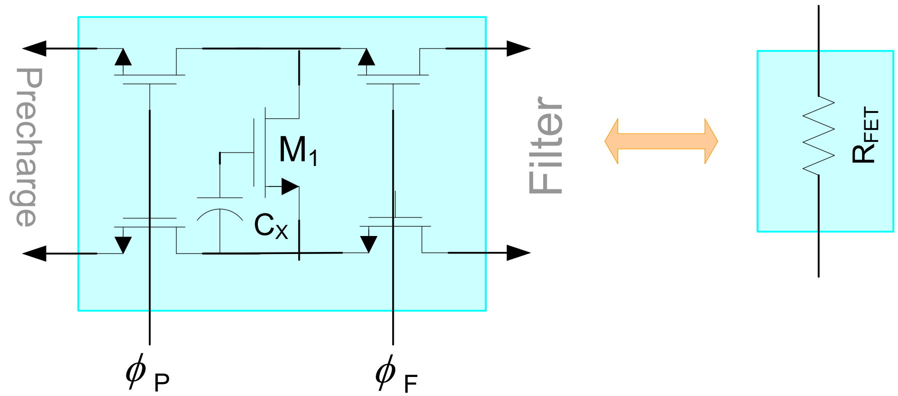
Sometimes termed “current mode”

- Switched Capacitor
- Switched Resistor

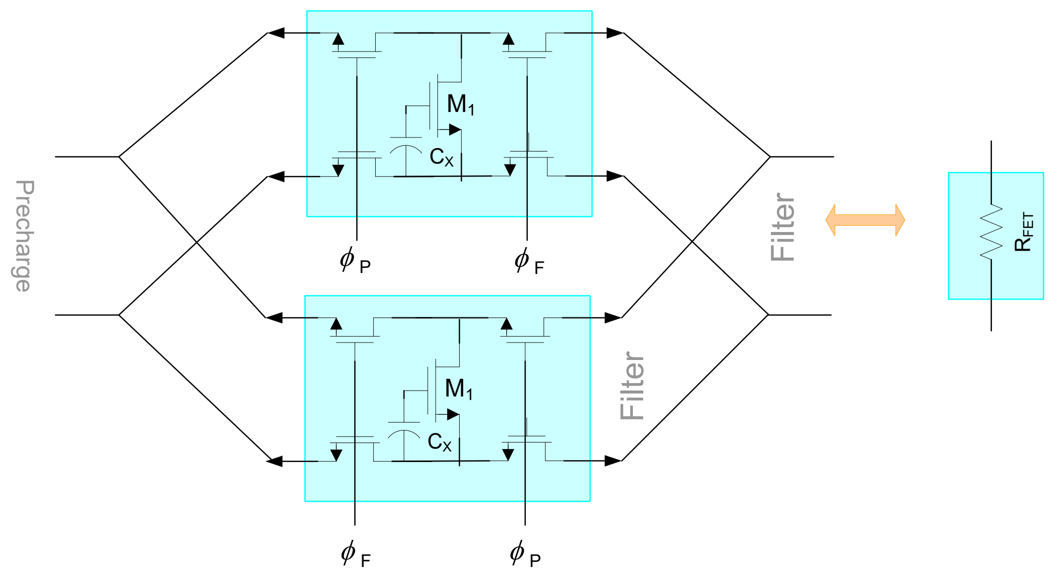
Will discuss later

- Other Structures

# Switched-Resistor Voltage Mode  Integrators



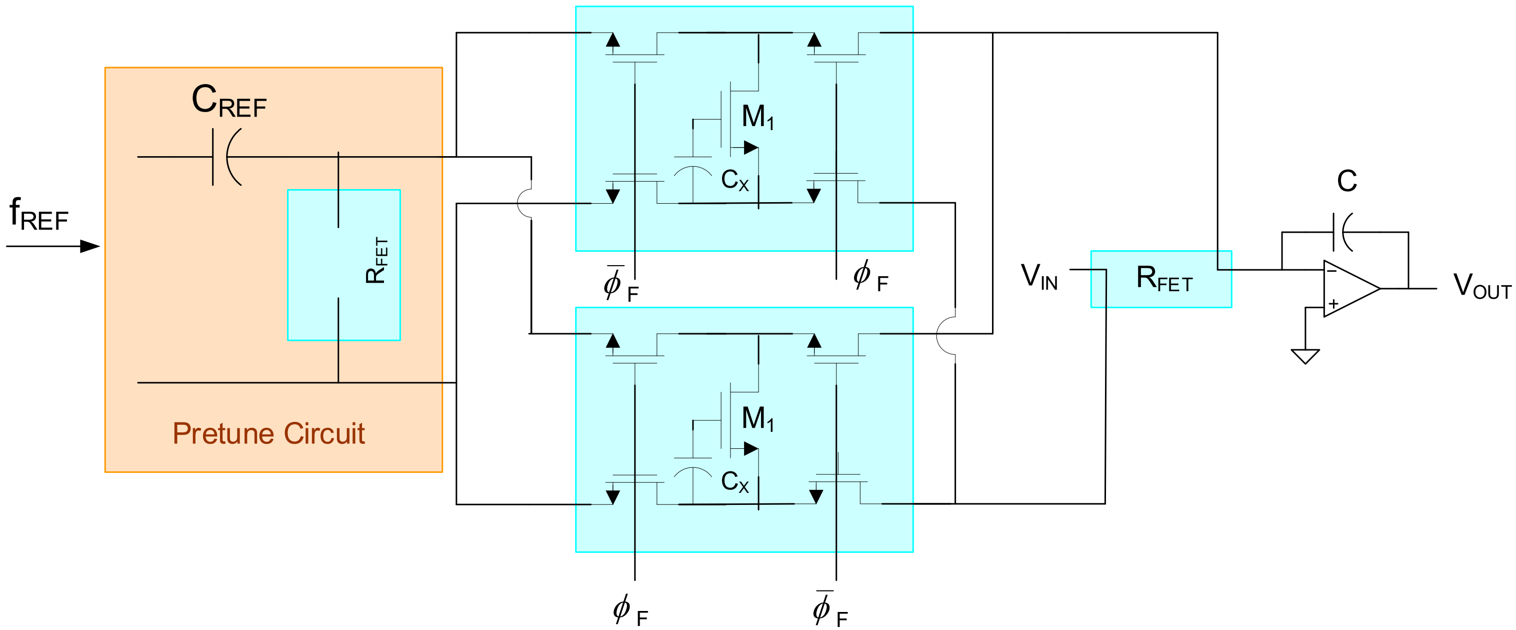

Observe that if a triode-region MOS device  is switched between a precharge circuit and a filter circuit (or integrator) and $V_{GS}$ is held constant,   It will behave as a resistor while in the filter circuit




$$R_{FET} \cong \frac{L}{\mu C_{OX} W (V_{GS} - V_T)}$$

Observe that if two such circuits are switched between a precharge circuit and a filter circuit (or integrator) and $V_{GS}$ is held constant, it will behave as a resistor in the filter circuit at all times

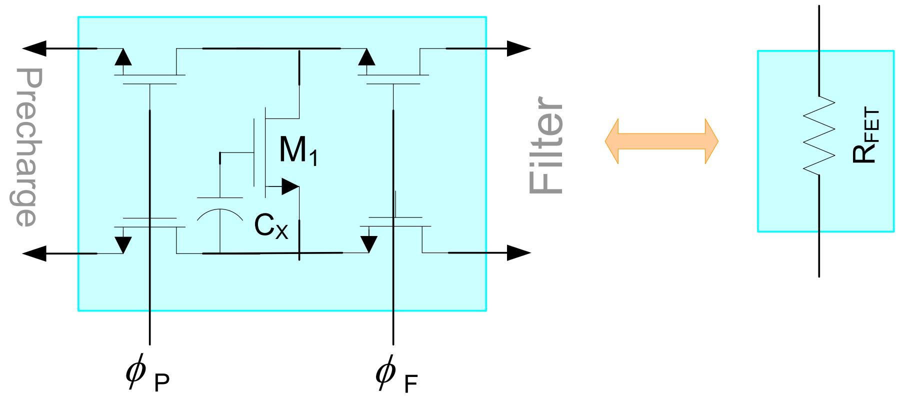
# Switched-Resistor Voltage Mode Integrators



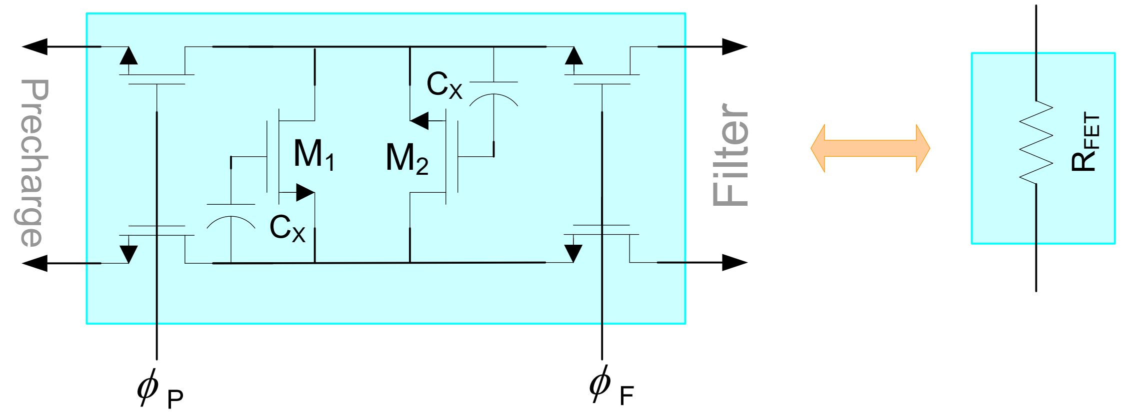

Switched-resistor integrator

- Clock frequency need only be fast enough to prevent droop on $C_X$
- Minor overlap or non-overlap of clock plays minimal role in integrator performance
- Switched-resistors can be used for integrator resistor or to replace all resistors in any filter
- Pretune circuit can accurately establish $R_{FET}C_{REF}$ product proportional to $f_{REF}$
- $R_{FET}C$ product is given by $R_{FET}C = R_{FET}C\frac{C_{REF}}{C_{REF}} = [R_{FET}C_{REF}]\bullet\left[\frac{C}{C_{REF}}\right]$ and is thus accurately controlled

# Switched-Resistor Voltage Mode Integrators

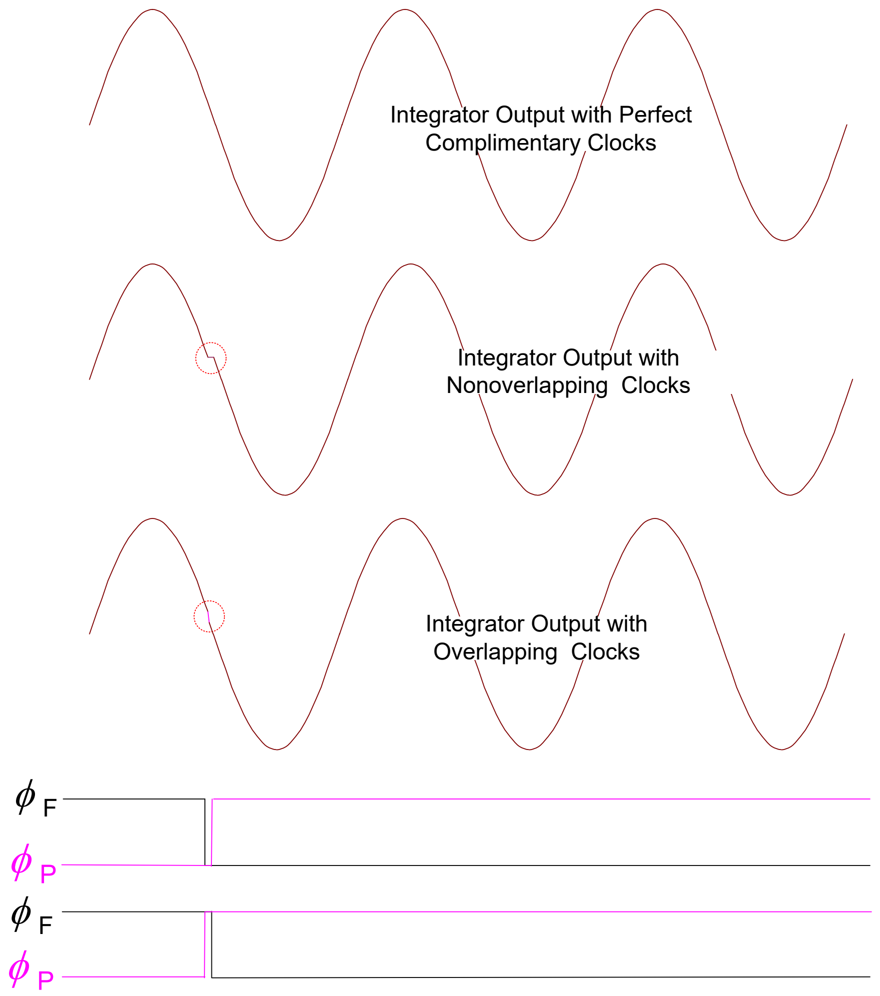



There are some modest nonlinearities in this MOSFET when operating in the triode region



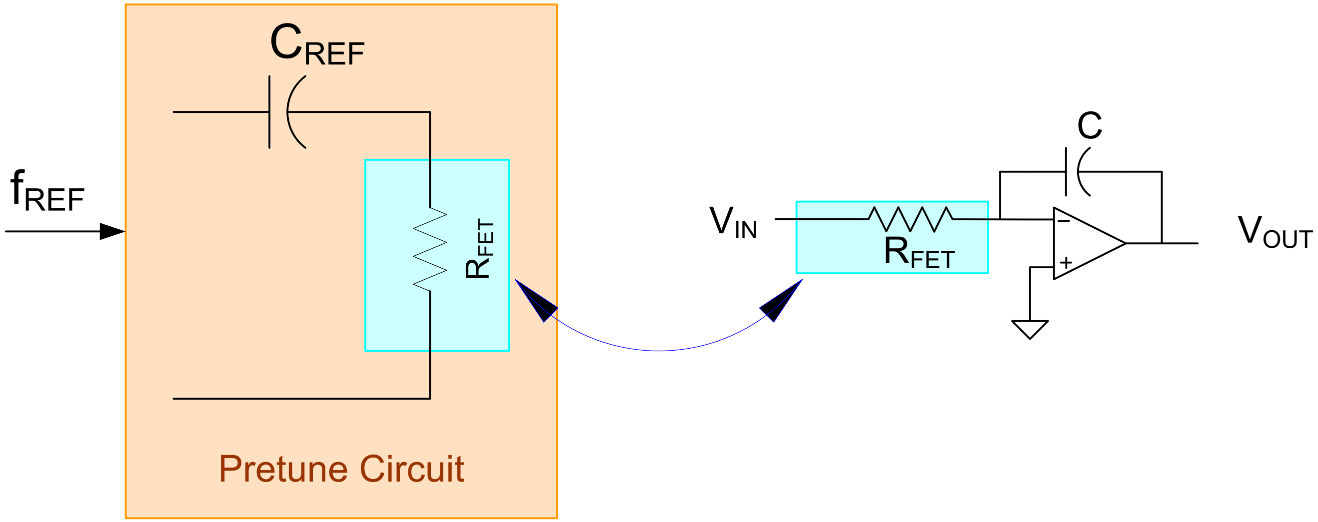

- Significant improvement in linearity by cross-coupling a pair of triode region resistors
- Perfectly cancels nonlinearities if square law model is valid for $M_1$ and $M_2$
- Only modest additional complexity in the Precharge circuit

# Switched-Resistor Voltage Mode Integrators




- Aberrations are very small, occur very infrequently, and are further filtered
- Play almost no role on performance of integrator or filter

# Switched-Resistor Voltage Mode Integrators




Switched-resistor integrator

- Accurate $CR_{FET}$ products is possible
- Area reduced compared to Active RC structure because $R_{FET}$ small
- Single pretune circuit can be used to “calibrate” large number of resistors
- Clock frequency not fast and not critical (but accuracy of $f_{REF}$ is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

# Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

} Sometimes termed "current mode"

- Switched Capacitor (Feedback-based)
- Switched Resistor (Feedback-based)

} Discrete Time

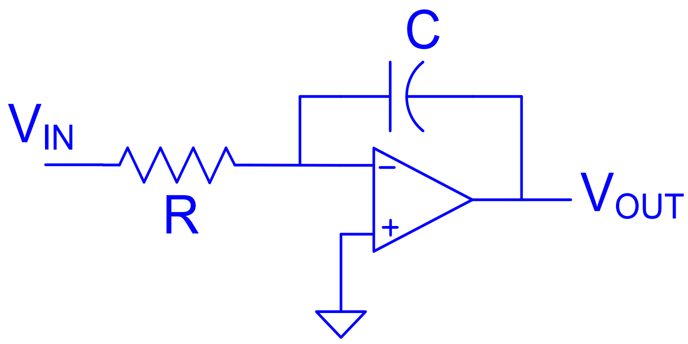
- Other Structures

Have introduced a basic voltage-mode integrators in each of these approaches

All of these structures have applications where they are useful

Performance of feedback-based structures limited by Op Amp BW

# Variants of basic inverting integrator have been considered




Basic Miller Integrator

- Active RC
- MOSFET-C
- OTA-C
- $g_m$-C
- Switched-Capacitor
- Switched-Resistor

**Performance of all is limited by GB of Operational Amplifiers**

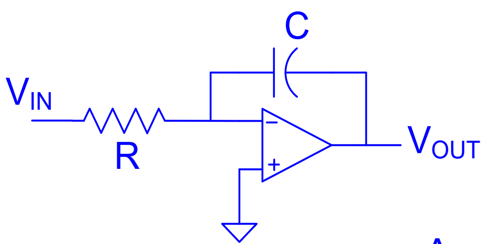
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

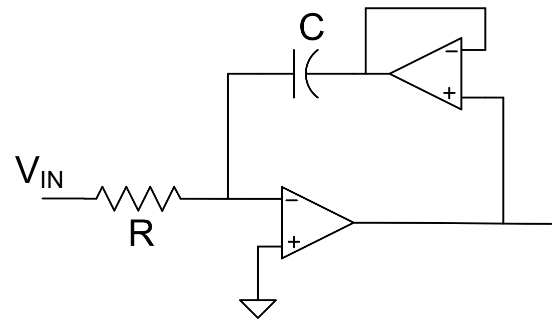
Need metric for comparing integrator performance

# Are there other integrators in the basic classes that have been considered?



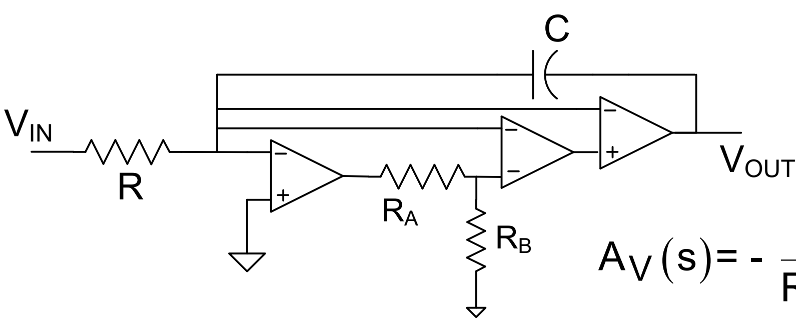

$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting



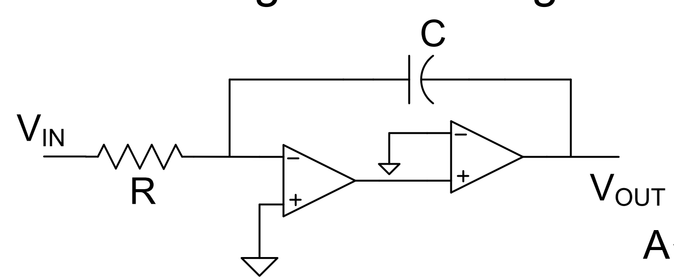

$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting



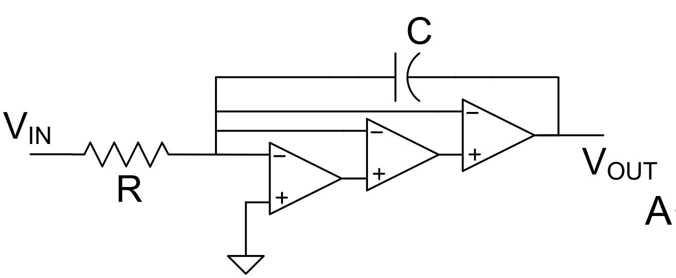

$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



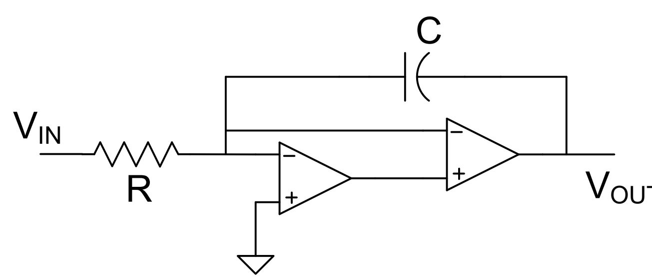

$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting




$$A_V(s) = -\frac{1}{RCs}$$

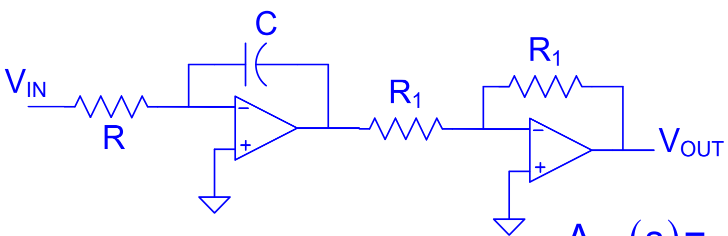
Zero Second Derivative Inverting




$$A_V(s) = -\frac{1}{RCs}$$

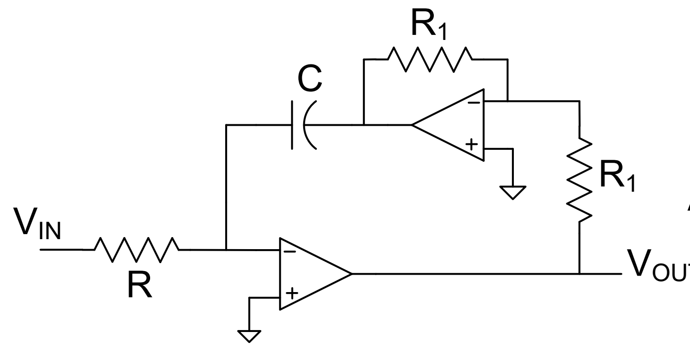
Zero Sensitivity Inverting

# Are there other integrators in the basic classes that have been considered?



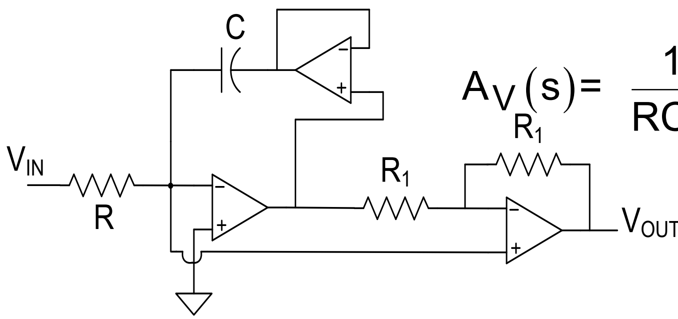

Miller Noninverting

$$A_V(s) = \frac{1}{RCs}$$



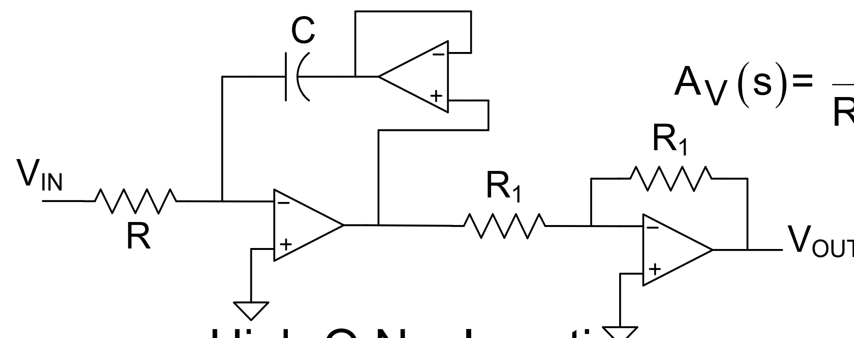

$$A_V(s) = \frac{1}{RCs}$$

Phase Lead Integrator



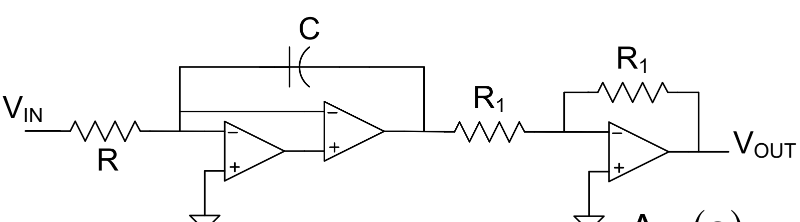

$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting



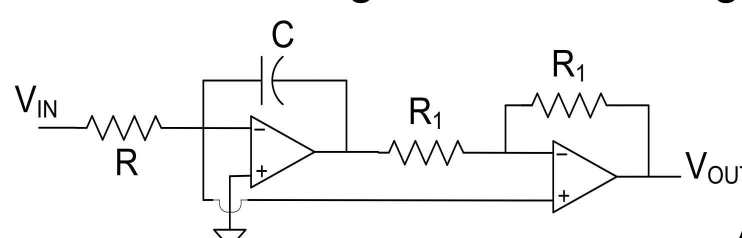

$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting




$$A_V(s) = \frac{1}{RCs}$$

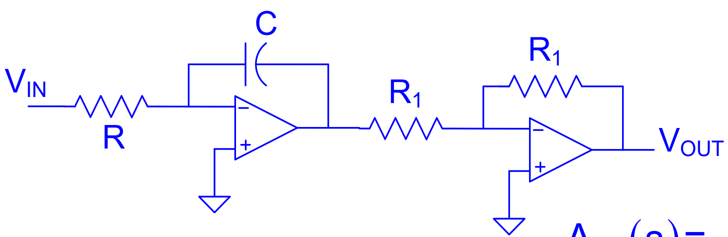
Zero Sensitivity Noninverting




$$A_V(s) = \frac{1}{RCs}$$

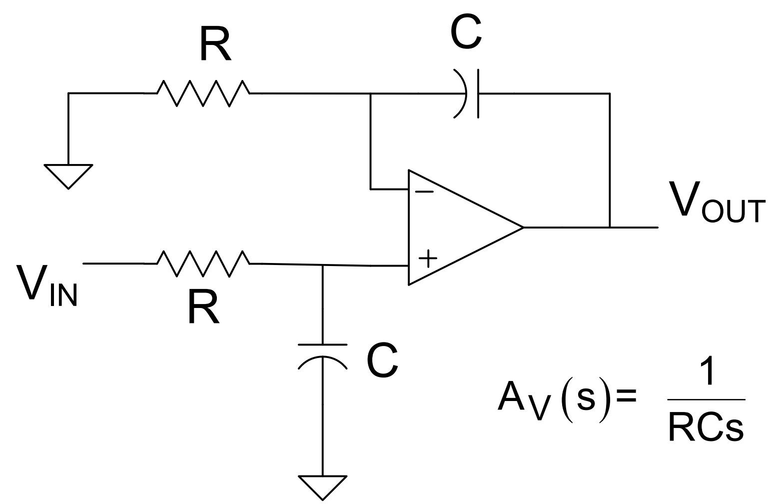
Modified Miller Noninverting

# Are there other integrators in the basic classes that have been considered?



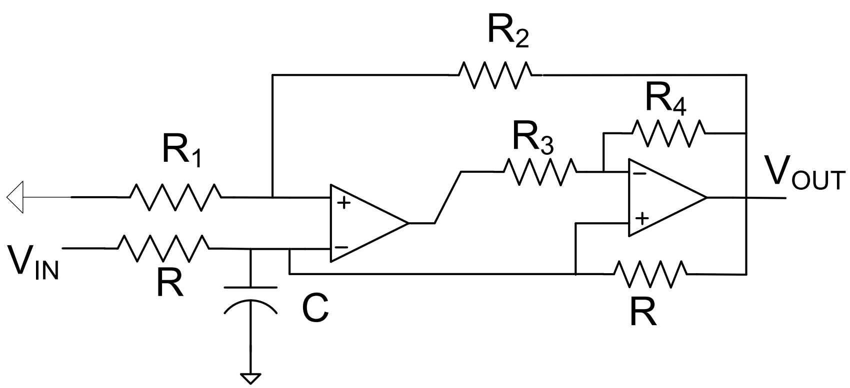

Miller Noninverting

$$A_V(s)=\frac{1}{RCs}$$




$$A_V(s)=\frac{1}{RCs}$$

Balanced Time Constant Noninverting




Zero Sensitivity Noninverting

$$A_V(s)=\frac{2}{RCs}$$

If $R_1=R_2$ and $R_3=R_4$

(note this has a grounded integrating capacitor!)

Stay Safe and Stay Healthy !

End of Lecture 27